B. Sc - Part - II Paper III

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Group theory - Group - Let G be a non-empty set together with binary operation 'o' is said to be a group if it satisfied the following laws

1.) **Closure property** - If $a \in G$ and $b \in G$ then $a o b \in G$

2.) **Associativity** - If $a, b, c \in G$ then

$$(a o b) o c = a o (b o c) \quad \forall a, b, c \in G.$$

3.) **Existence of identity** - There exists an element $e \in G$ such that $ea = a = ae \quad \forall a \in G$. The element e is called identity.

4.) **Existence of inverse** - For each element a in G, there is an element b in G (called an inverse of a) such that $a o b = b o a = e$.

Abelian Group OR Commutative Group:

A group G is said to be abelian or commutative if it satisfied commutativity i.e $a o b = b o a$ $\forall a, b \in G$.

Example of Group

1.) Set of integers i.e I is a group w.r.t. the operation of addition of integers.

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Example 2

Set of all non-zero rational numbers forms a Group under the operation of multiplication of rational numbers.

Example of Abelian group –

The set of all positive rational numbers forms an abelian group under the composition defined by $a \circ b = ab/2$.

Finite Group If a group G consists of a finite number of distinct elements then the group is called finite group otherwise an infinite group.

$U(n)$ is an example of finite group.

Note - $U(n)$ is the set of all positive integers less than n and relatively prime to n. Then $U(n)$ is a group under multiplication modulo n is an example of finite group.

Example $U(10) = \{1, 3, 7, 9\}$ is an example of finite abelian group.

$\Rightarrow$ Set of integer under addition is an example of infinite group.

Properties of Groups

Th.1 Uniqueness of identity – The identity element in a group is unique.

Proof: Suppose e and e' are two identity elements of a group.

Then $e \circ e' = e$ if e' is ~~identity~~ identity.

$e' \circ e = e'$ if e is identity.

But $e \circ e'$ is a unique element of G.

$\therefore ee' = e$ and $ee' = e' \Rightarrow e = e'$

Hence the identity element is unique.

Th. 2. Uniqueness of inverse The inverse of each element of a group is unique.

Proof! – Let a be any element of group G.

Let e be the identity element.

Suppose b and c are two inverse of a i.e.

$b \circ a = e = a \circ b$ and $c \circ a = e = a \circ c$.

We have $b \circ (a \circ c) = b \circ e$
$= b$

Also $(b \circ a) \circ c = e \circ c$
$= c$

But in a group composition is associative.

Therefore $b \circ (a \circ c) = (b \circ a) \circ c$ Hence $b = c$.